# 2021

## STATISTICS—GENERAL

Paper: GE/CC-3

#### (Introduction to Statistical Inference)

Full Marks: 50

The questions are of equal value.

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

# **1.** Answer *any five* questions:

 $2 \times 5$ 

- (a) If  $X_1, X_2, X_3$  are *i. i. d.* N(0,1) variates, write down the p.d.f of  $T = \frac{\sqrt{2}X_1}{\sqrt{X_2^2 + X_3^2}}$
- (b) Distinguish between estimator and estimate.
- (c) In a Bernoulli distribution with parameter p,  $H_0$ :  $p = \frac{1}{3}$  is rejected if more than 3 heads are obtained out of 5 throws of a coin. Find the probability of Type I error.
- (d) For a one-way ANOVA, find an unbiased estimator of the error variance.
- (e) Let  $(X_1, X_2, ..., X_n)$  be a random sample from  $N(\mu, 1)$  population. Find the maximum likelihood estimate of  $\mu$ .
- (f) Write one disadvantage of a CRD.
- (g) What is a BLUE?
- (h) Define power of a test.

# 2. Answer *any two* questions:

5×2

- (a) If  $X_1$  and  $X_2$  are independently distributed Poisson random variables with parameters  $\lambda_1$  and  $\lambda_2$  respectively, obtain the distribution of  $X_1 + X_2$ .
- (b) If  $(X_1, X_2, ..., X_n)$  is a random sample from normal population  $N(\theta, 1)$ , show that  $\bar{X}^2 \frac{1}{n}$  is an unbiased estimator of  $\theta^2$ .
- (c) Derive the distribution of the different sums of squares involved in a one-way classified data.

# 3. Answer any three questions:

 $10 \times 3$ 

- (a) Suppose  $(X_1, X_2, ..., X_n)$  be a random sample from  $N(\mu, \sigma^2)$  distribution. Derive the maximum likelihood estimators of  $\mu$  and  $\sigma^2$ . Also examine the consistency of the sample mean as an estimator of  $\mu$ .
- (b) Describe a suitable test for testing  $H_0: \mu_1 = \mu_2$  against all possible alternatives for two independent normal populations  $N(\mu_1, 3^2)$  and  $N(\mu_2, 4^2)$ . Also find a 95% confidence interval for  $\mu_1 \mu_2$ .

Please Turn Over

- (c) If  $X_1, X_2, X_3, X_4$  are independent random variables each distributed as normal with zero mean and variance 4. Find the distributions of 5+5
  - (i)  $X_1 + X_2$
  - (ii)  $\frac{X_1^2 + X_4^2}{X_2^2 + X_3^2}$
- (d) Briefly describe the analysis of variance technique for analyzing a two-way classified data with only one observation in each cell. Clearly state all the assumptions.
- (e) (i) Discuss any one of the basic principles of design of experiment.
  - (ii) Give the layout of an RBD.

5+5