

2021

## PHYSICS — HONOURS

Paper : CC-12

Full Marks : 50

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.***[Syllabus : 2019-20]****(Statistical Physics)**Answer **question no. 1** and **any four** questions from the rest.1. Answer **any five** questions :

2×5

- (a) Show that for a canonical system :  $\overline{(E - \bar{E})^2} = kT^2 C_V$ , where  $C_V$  is the heat capacity at constant volume.
- (b) A particle of mass  $M$  is falling freely under gravity starting from rest. Draw its phase trajectory.
- (c) The entropy of black body radiation is given by  $S = \frac{4}{3} \sigma V^{\frac{1}{4}} E^{\frac{3}{4}}$ . Show that  $PV = \frac{E}{3}$ .
- (d) Consider a free particle inside a 1D box of length  $L$ . Calculate the number of microstates between the energy values  $E$  and  $E + dE$ .
- (e) Can  ${}^7_3\text{Li}$  form BEC? Give reason.
- (f) In how many ways can 5 identical balls be distributed among 3 identical boxes where each box can contain any number of balls?
- (g) Three containers, each of volume  $V$ , contain  $N$  particles of a classical, a Bose and a Fermi gas respectively at the same temperature  $T$ . State with reason which of the three containers will have the highest pressure.
2. (a) What is the phase trajectory of a simple pendulum performing small oscillations? Show that the area enclosed by the trajectory is equal to the product of the total energy  $E$  and the time period  $T$  of the pendulum.
- (b) Energy of a particle in 1D has the form  $E = ap^2 + bq^5$  where  $p$  and  $q$  are the generalised momentum and coordinate and 'a' and 'b' are constants. Calculate the specific heat. (2+3)+5

Please Turn Over

3. A system of  $N$  classical particles in thermal equilibrium are distributed between two energy levels  $\epsilon = -\Delta/2$  and  $\epsilon = \Delta/2$ .
- Write down the partition function for the system.
  - Calculate the internal energy and entropy of the system.
  - What is the specific heat of the system?
  - Plot the specific heat and the entropy as a function of temperature and explain the high temperature and low temperature behaviour of the curves. 2+(1+1)+2+(2+2)
4. (a) Find the variation of the specific heat  $C_V$  as a function of the temperature  $T$  for photon gas confined in 1D box.
- (b) Given the energy of a system at temperature  $T$  and volume  $V$  is
- $$E = aT^4V$$
- where 'a' is a constant. Calculate (i) entropy (ii) Helmholtz free energy and (iii) Gibb's free energy. 5+(1+2+2)
5. For a classical ideal gas, derive the equation of state separately using (a) Canonical partition function and (b) Grand canonical partition function. 5+5
6. (a) Consider a photon gas confined in a volume  $V$  at temperature  $T$ . Show that the number of photons in this volume is proportional to  $T^3$ .
- (b) A photon gas is confined in volume  $V$  at temperature  $T$ . If the volume is increased adiabatically to  $2V$ , determine the final temperature.
- (c) Derive Wein's displacement law from Planck's law. 3+2+5
7. (a) Sketch the Fermi-Dirac distribution function and its derivative for  $T = 0$  K and  $T > 0$  K showing clearly the Fermi energy.
- (b) Explain physically how the electronic specific heat of a metal behaves as a function of temperature.
- (c) Deduce the pressure-volume relationship for a free electron gas obeying Fermi-Dirac statistics at 0K. Hence find an expression for the bulk modulus of the gas. 3+3+4
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**[Syllabus : 2018-19]****(Solid State Physics)**

Answer *question no. 1* and *any four* questions from the rest.

1. Answer *any five* questions :

2×5

- Sketch (210) and  $(\bar{1}\bar{1}\bar{1})$  planes of a cubic system.
- Determine the relationships between the lattice parameter 'a' and the atomic radius 'r' for monoatomic simple cubic, bcc and fcc structures.
- Show that the reciprocal lattice to a simple cubic lattice is also a simple cubic lattice with lattice constant  $\frac{2\pi}{a}$ .
- Explain hysteresis for ferroelectric materials.
- Give an indirect evidence for the existence of phonons.
- Consider two ferromagnets : one having a hysteresis curve with broad area and another with a narrow area. Which one can be used as electromagnet and why?
- The atomic polarizability of neon is  $4.3 \times 10^{-41} \text{ Fm}^2$ . If a neon atom is placed in an electric field of  $5 \times 10^6 \text{ V/m}$ , calculate its dipole moment and the displacement of the centroids of positive and negative charges in it.

2. (a) In a cubic crystal, show that the distance between the adjacent planes with Miller indices  $hkl$  is given by

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} .$$

- Considering the scattering of X-rays from individual atoms in a crystal followed by their recombination to obtain directions of diffraction maxima derive the Laue equations.
- An X-ray analysis of a crystal is made with monochromatic X-rays of wavelength  $0.58 \text{ \AA}$ . Bragg's reflections are obtained at angles of (i)  $6.45^\circ$  (ii)  $9.15^\circ$  and (iii)  $13^\circ$ . Calculate the interplanar spacing of the crystal. 2+4+4

3. (a) Discuss the failure of classical theory in explaining the observed temperature dependence of specific heat of a solid.

(b) Calculate the Debye frequency for aluminium from the following data :

Density of atoms in Al =  $6.02 \times 10^{28} / \text{m}^3$ ,  $v_l = 6374 \text{ m/s}$ ,  $v_t = 3111 \text{ m/s}$ .

**Please Turn Over**

- (c) What are phonons? What is the physical significance of Debye temperature? Consider the expression for internal energy of a lattice in Debye model :

$$U = 9R \frac{T^4}{\Theta_D^3} \int_0^{\frac{\Theta_D}{T}} \frac{x^3 dx}{e^x - 1} \quad (\text{where the symbols have their usual meanings})$$

Obtain an expression for the specific heat  $C_V$  at low temperature. What will happen to  $C_V$  at high temperature? 3+2+(1+2+2)

4. (a) Suppose a paramagnetic atom having permanent moment  $\vec{\mu}$  with a given resultant quantum number  $\vec{J}$  is placed in a uniform magnetic field  $\vec{B}$ . Obtain an expression of the magnetization as a function of  $\vec{B}$  and temperature  $T$ . Hence, obtain Curie's law in the appropriate limit.
- (b) Show that the force exerted by a field gradient on a specimen is proportional to its paramagnetic susceptibility.
- (c) Explain why diamagnetism is an inherent property of an atom. (4+2)+3+1

5. (a) The dispersion relation of electrons in a 3d lattice is given by

$$\varepsilon(k) = \alpha \cos k_x a + \beta \cos k_y a + \gamma \cos k_z a$$

where  $a$  is the lattice constant and  $\alpha, \beta, \gamma$  are constants. Find the effective mass tensor at the corner of the first Brillouin zone  $\left(\frac{\pi}{a}, \frac{\pi}{a}, \frac{\pi}{a}\right)$ .

- (b) Calculate the Hall coefficient  $R_H$  in a solid where both electrons and holes contribute to the Hall effect.
- (c) Schematically represent the variation of velocity, effective mass and acceleration as a function of wave vector. 4+3+3
6. (a) What do you mean by orientational polarization of molecules? Discuss the temperature dependence of such polarization.
- (b) What do you mean by plasma frequency of free electrons? Using Lorentz model, derive Sellmeyer's equation for elastically bound electrons.
- (c) What is the origin of piezoelectric effect? Mention one application of piezoelectric phenomenon. (1+2)+(1+3)+(2+1)

7. (a) What does the existence of energy gap in a superconductor imply?
- (b) What is the relation between isotopic mass and transition temperature in a superconductor? Show the variation of energy gap with temperature.
- (c) Write down the expression for penetration of external magnetic field inside a superconductor.
- (d) In an experiment, a niobium (Nb) wire of radius 0.25 mm is immersed in liquid helium ( $T = 4.2$  K) and required to carry a current of 300 A. It is given that  $H_C(0) = 0.20$  T and the critical transition temperature  $T_C$  of Nb is 9.3 K. Will the wire remain superconducting? 2+(2+2)+1+3