

2021

## ADVANCED BUSINESS MATHEMATICS — HONOURS

First Paper

(C-21-A)

Full Marks : 50

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*

## Group - A

1. Answer the following questions :

(a) If  $A = \begin{bmatrix} 6 & 7 & 5 \\ 1 & 0 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 7 & 5 \\ 4 & 0 & 6 \end{bmatrix}$ , then find the matrix  $2A - B$ . 2

*Or,*

Given  $A = \begin{bmatrix} a & b \\ 1 & 0 \end{bmatrix}$  and  $AA^T = I_2$ , then find  $a$  and  $b$ . 2

(b) If  $f(x) = 2x^2 - 5x + 4$ , for what value of  $x$  is  $2f(x) = f(2x)$ ? 2

(c) If  $f(x+3) = 3x^2 - 2x + 5$ , find  $f(x-1)$ . 2

(d) Evaluate :  $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$ . 2

*Or,*A function  $f(x)$  is defined as follows :

$$f(x) = 2x - 4 \quad \text{when } x \geq 3$$

$$= 2 \quad \text{when } x < 3.$$

Examine whether  $\lim_{x \rightarrow 3} f(x)$  exists or not. 2

(e) Integrate (*any one*) :

(i)  $\int \frac{x}{x-1} dx$

(ii)  $\int \frac{1}{2x+1} dx$ . 2

Please Turn Over

**Group - B**

2. Answer the following :

(a) Evaluate **any two** :

3×2

(i)  $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + a} - \sqrt{a - x^2}}{x^2}$

(ii)  $\lim_{x \rightarrow 4} \left[ \frac{1}{x+4} + \frac{8}{x^2 - 16} \right]$

(iii)  $\lim_{x \rightarrow \infty} \frac{15x^7 + 12x + 17}{5x^7 + 9x^2 + 12}$

(b) If  $x\sqrt{1+y} + y\sqrt{1+x} = 0$ , prove that  $\frac{dy}{dx} = \frac{-1}{(1+x^2)}$ .

6

**Or,**

If  $x^m y^n = (x+y)^{m+n}$ , show that  $\frac{dy}{dx} = \frac{y}{x}$ .

6

(c) If  $x + y = 2$ , show that the maximum value of  $\left(\frac{4}{x} + \frac{36}{y}\right)$  is less than its minimum value.

6

(d) If  $x + y + z = 0$ , show that  $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = 0$ .

6

(e) Solve the following equations by matrix inverse method :

$3x - y + 2z = 7, x + 2y - z = 3, x + y + z = 4.$

6

**Group - C**

Answer **any one** question.

3. (a) If  $f(x) = \log_e \left( \frac{1-x}{1+x} \right)$ , then prove that  $f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)$ .

(b) Find the area bounded by the straight line  $y = 3x$ , the  $x$ -axis and the ordinates  $x = 1, x = 2$ . 4+6

4. (a) Evaluate :  $\int_1^2 \log x \, dx$ .

(b) If  $V = \log \frac{x^3 + y^3}{x^2 + y^2}$ , then show that  $x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y} = 1$ .

4+6