## 2021

## MATHEMATICS - GENERAL

## Paper : GE/CC-4

Full Marks : 65
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words
as far as practicable.

## Group - A

1. Choose the correct answer :
(a) In the group of all odd integers with respect to the composition * defined by $a * b=a+b-3$, the identity element is
(i) 0
(ii) 1
(iii) 2
(iv) 3 .
(b) If in some bivariate data, $\operatorname{Cov}(x, y)=32, \operatorname{Var}(x)=36, \operatorname{Var}(y)=64$, then the correlation coefficient of $x, y$ is
(i) $\frac{3}{2}$
(ii) $\frac{2}{3}$
(iii) $\frac{1}{24}$
(iv) 24 .
(c) The binary representation of the decimal number 34.25 is
(i) $100010 \cdot 01$
(ii) $100100 \cdot 01$
(iii) $100110 \cdot 01$
(iv) None of these.
(d) The decimal representation of the octal number 74.22 is
(i) 60.28215
(ii) 60.28521
(iii) 60.28125
(iv) None of these.
(e) Method which uses a list of well defined instructions to complete a task starting from a given initial state to end state is called as
(i) Program
(ii) Flowchart
(iii) Algorithm
(iv) Pseudo code.
(f) The full form of RAM is
(i) Real Access Memory
(ii) Random Access Memory
(iii) Running Action Memory
(iv) Real Action Memory.
(g) For any two events A and B , it is given that $\mathrm{P}(\mathrm{A})=0.6, \mathrm{P}(\mathrm{AB})=0.3$ and $\mathrm{P}(\mathrm{A} / \mathrm{B})=0.75$, then $P\left(B^{C}\right)$ is equal to
(i) 0.40
(ii) 0.75
(iii) 0.25
(iv) 0.60 .
(h) A random variable $X$ has probability density function $f(x)=\left\{\begin{aligned} 1 / 4, & -2<x<2 \\ 0, & \text { elsewhere }\end{aligned}\right.$ then $P(x<1)=$
(i) $1 / 4$
(ii) $1 / 2$
(iii) $3 / 4$
(iv) None of these.
(i) Probability that the exactly one of the events $A$ and $B$ occurs is
(i) $\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})+1$
(ii) $\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})-1$
(iii) $\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})$
(iv) None of these.
(j) If the assumed hypothesis is tested for rejection considering it to be true is called
(i) Simple hypothesis
(ii) Composite hypothesis
(iii) Null hypothesis
(iv) Alternative hypothesis.

## Group - B <br> Unit - 1 <br> (Algebra-II)

2. Answer any three questions:
(a) Let $P$ be the set of real numbers except the integer 1 . Let the operation ' $*$ ' in $P$ be defined by, $a * b=a+b-a b$, for all $a, b \in P$. Show that $(P, *)$ is a group.
(b) Let $S=\{a+b \sqrt{5}: a, b$ are rational numbers $\}$. Show that $S$ is subfield of the field $(R,+,$.$) of real$ numbers.
(c) Find the eigenvalues and the corresponding eigenvectors of the matrix $\left(\begin{array}{lll}2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2\end{array}\right)$.
(d) Show that the real quadratic form $x^{2}+2 y^{2}+2 z^{2}+2 x y+2 z x$ is positive semi-definite.
(e) Use Cayley-Hamilton theorem to compute the inverse of the matrix $\left(\begin{array}{rr}3 & 1 \\ -1 & 2\end{array}\right)$.

## Unit - 2

## (Computer Science and Programming)

3. Answer any four questions:
(a) Draw a flowchart to find all the odd numbers from 1 to 100 and compute their sum.
(b) Write a FORTRAN program to calculate the perimeter, area and diagonal of a rectangle whose two adjacent sides are $a$ and $b$.
(c) Obtain the binary equivalent of the numbers $(34.625)_{10}$ and $(0.875)_{10}$. Find their product and difference in binary number system. Find the octal equivalent of the product.
(d) Draw a flowchart to find the sum of the following series :

$$
1+\frac{1}{3^{2}}+\frac{1}{5^{2}}+\ldots \ldots \ldots .+\frac{1}{51^{2}}
$$

(e) (i) Convert the expression $\frac{(a+b)^{2}}{(c+4 d)^{2} b}$ into FORTRAN code.
(ii) Compute in FORTRAN if $\mathrm{K}=-4, \mathrm{~L}=7, \mathrm{M}=9, \mathrm{~N}=5,(\mathrm{~K}+\mathrm{L})^{*} \mathrm{M} / \mathrm{N}+\mathrm{L}=$.
(f) Write a FORTRAN program to compare any two numbers (i.e. whether one is greater than or less than or equal to other one) using if else statement.
(g) Write a FORTRAN program which will take temperature of a city in Centigrade degrees as input and convert the same in Fahrenheit degrees.

## Unit - 3 <br> (Probability and Statistics)

4. Answer any four questions:
(a) A speaks the truth in $60 \%$ of the cases and B is $90 \%$ of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?
(b) In a bolt factory machines A, B and C manufacture 25,35 and 40 percentage of the total production respectively. Of these outputs 5, 4 and 2 percentage are defective bolts. A bolt is drawn at random from the product and is found defective. What is the probability that it was manufactured by A?
(c) Determine the value of $K$, such that $f(x)$ defined by $f(x)=\left\{\begin{array}{ll}K x(1-x), & 0<x<1 \\ 0, & \text { elsewhere }\end{array}\right.$ is a probability density function. Find also the corresponding distribution function.
(d) Find the two regression equations from the following data:

| $x$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | 2 | 3 | 5 | 4 | 6 |

(e) Let $P$ denote the probability of getting a head when a given coin is tossed once. Suppose that the hypothesis $H_{0}: P=\frac{1}{2}$ is rejected in favour of $H_{1}: P=\frac{3}{4}$ if more than 3 heads are obtained out of 5 throws of a coin. Find the probability of Type-I and Type-II error.
(f) The two regression lines involving two variables $x$ and $y$ are $y=5.6+1.2 x$ and $x=12.5+0.6 y$. Find the mean of $x, y$ and their correlation coefficient.
(g) Out of 400 fruits selected at random from a large population 53 were found to be bad. Test at $1 \%$ significance level that on the average $10 \%$ of the fruits were bad.
[Given that $\frac{1}{\sqrt{2} \pi} \int_{2.58}^{\infty} e^{-\frac{x^{2}}{2}} d x=0.005$ ]

