2021

MATHEMATICS — HONOURS

Paper : SEC-B-2

(Scientific Computing with SageMath/R)

Full Marks : 80

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

(Answer either Scientific Computing with SageMath or Scientific Computing with R)

[Scientific Computing with SageMath]

Notations and symbols have their usual meaning

Throughout the question paper sage code means code in SageMath.

1. Choose the correct alternative (Give justification whenever it is applicable) : 2×10

(a) What is the correct code in Sage to evaluate the value of π correct up to 100 digits?

(i) $n(\pi, 100)$ (ii) n(pi, digits=100)

(iii) $N(\pi,$ prec=100) (iv) N(pi,100).

(b) What will be the result of the following Sage code: exp(11*8+5) – e^(11*8+5)?

(i) 0 (ii) error: "exp" is not defined

(iii) error: "e" is not defined (iv) error: "e^" is not defined.

(c) What will be your code in Sage to find the natural logarithm of 100 in decimal approximation?

(i) $N\big(\log(100)\big)$ (ii) $n\big(\log(100,10)\big)$

(iii) $n\big(\log\exp(100)\big)$ (iv) $N\big(\log e(100)\big)$.

(d) What will be your code in Sage, if you want all the square roots of 4?

(i) Sqrt(4) (ii) sqrt(4)

(iii) sqrt(4, all==true) (iv) sqrt(4, all=true).

(e) The output of the Sage code : 4*(10//4) + 10%4 == 10, 3*3<3 is

(i) (True, False) (ii) (True, True)

(iii) (False, True) (iv) (False, False).

(f) Which of the following is NOT the correct option for a^{b^c} in sage?

(i) $a \wedge b \wedge c$ (ii) $a**b**c$

(iii) $a \wedge\wedge b \wedge\wedge c$ (iv) $a \wedge b**c$.

(g) What will be the output of the following sage code?

```
a=4
b=2
print(a+b*2)
```

(i) 36 (ii) 10

(iii) 8 (iv) 12

(h) What will be the output of the following sage code?

```
a =[1, 3]
b=[10, 20, a]
print(b)
```

(i) $[10, 20, [1,3]]$ (ii) [10, 20, 1, 3]

(iii) An error will occur (iv) [1, 3, 10, 20]

(i) What is the correct code in Sage to plot the function x^3-x in $-2 < x < 2$?

(i) plot(x^3 – x, –2, 2) (ii) $\text{plot}\left(x^3 - x, -2, 2\right)$

(iii) $\text{plot}\left(x\text{^}3 - x, \text{range}[-2, 2]\right)$ (iv) $\text{plot}\left(x\text{^}3 - x; -2\ 2\right)$

(j) What will be the code in Sage to compute $\int_0^1 \frac{x}{x^2+1} dx$?

(i) $\text{integral}\left(x/\left(x\text{^}2+1\right), x, 0, 1\right)$ (ii) $\text{integration}\left(x/\left(x\text{^}2+1\right), x, 0, 1\right)$

(iii) $\text{integral}\left(x/\left(x\text{^}2+1\right), x; 0, 1\right)$ (iv) $\text{integrate}\left(x/\left(x\text{^}2+1\right), 0, 1\right)$

Section - I

Answer ***any one*** question.

2. (a) What are open source softwares? "Sage is a free and open source mathematical software." — Explain in brief.

(b) Write sage codes to find the difference between golden ratio and $\frac{f_{42}}{f_{41}}$ using 20 digits where f_n is the *nth* term of the Fibonacci sequence. (2+4)+4

3. (a) What will be the output of the following sage commands? Explain your answer :

$y = 3; y = 3 * y + 1; y = 3 * y + 1; y$

(b) What will be the output of the following sage commands?

(i) 79/12

(ii) 79/12.0

(iii) 70//12

(c) Give the output of the following sage commands

$10 \wedge (1/2)$, sin(60) 4+(1+1+1)+(2+1)

Section - II

Answer ***any one*** question.

4. Write Sage code to plot the graphs of the following in one plot :

(a) graph of the function $f(x) = x^2 + 1$ in the range $(-2, 2)$.

(b) graph of the tangent line to the above curve : $f'(x) = 2x$ in the range $(-2, 2)$ with gridlines.

(c) one big dot at (1, 2) with size = 40. 4+4+2

5. (a) Write sage code to plot the graph of the function given by the polar curve

$r = 1 + 2 \cos 3\theta,\ 0 \le \theta \le 2\pi.$

(b) Write sage code to plot the graph of the following function :

$$f(x) = \begin{cases} 1+x, & -1 \le x \le 0 \\ 1+x^3, & 0 < x < 1 \end{cases}.$$

(c) Write sage code to plot the function $f(x) = \frac{1}{x} - x$ in $[-1, 1]$. Write also the equation of the asymptote (if it exists). 3+3+4

Section - III

Answer ***any one*** question.

6. Write sage code to do the following where $f(x) = x^3 + x$ 2×5

(a) find $f''(x)$.

(b) find $f'''(x)$.

(c) draw the graph of $f(x)$ where $-5 < x < 5$ with colour blue.

(d) draw the graph of $f''(x)$ where $-5 < x < 5$ with colour green.

(e) draw the graph of $f'''(x)$ where $-5 < x < 5$ with colour red.

(All the graphs should be in one plot).

Please Turn Over

7. Write codes in sage to do the following : 2×5

(a) Evaluate $\int x^n dx$

(b) Evaluate $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$

(c) Evaluate $\int_0^1 3x^2 dx$

(d) Evaluate $\int_0^\infty \frac{2}{\sqrt{\pi}} e^{-x^2} dx$

(e) Evaluate $\int_0^\infty \frac{\sin x}{x} dx$ to display 10 digits.

Section - IV

Answer ***any one*** question.

8. (a) Without using inbuilt functions write a program in sage to determine the total number of primes less than x, print the list of such primes and get an output for $x = 98$.

(b) Without using inbuilt function write a program in sage to determine factorial of 10. 5+5

9. (a) Without using inbuilt functions write a program in Sage to determine in decimal approximation the arithmetic mean and geometric mean of a list of numbers and get an output for the list 20, 15, 24, 31, 45, 17.

(b) Write the Sage codes (without using inbuilt functions) to find and print the median of the numbers 11, 23, – 34, 40, 50. 5+5

Section - V

Answer ***any one*** question.

10. (a) Define a matrix in sage whose rows are (1, – 3, 4, 7), (3, 4, 7, 9), (3, 7, 0, 11), (1, 3, – 4, 8). Give sage code to find the row reduced echelon form of the matrix.

(b) Write code in sage to find *det A* and A^{-1} where $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{pmatrix}$.

(c) Assuming the non-trivial solution exists, write sage commands to solve the following system of equations with matrices :

$$3x - 4y + 5z = 14$$
$$x + y - 8z = -5$$
$$2x + y + z = 7.$$

4+2+4

11. (a) Write sage code to solve the differential equation $\frac{dy}{dx} = \cos x$.

(b) Write sage commands to solve the following Initial Value Problem and plot the solution :

$$\frac{dy}{dx} = 1 - y,\ y(10) = 2$$

(c) Write sage commands to solve the following differential equation with the boundary conditions :

$$x_0 = 0,\ y(x_0) = 1,\ x_1 = \frac{\pi}{2},\ y(x_1) = 4$$

$$\frac{d^2y}{dx^2} + y = 0.$$

2+(2+2)+4

Section - VI

Answer ***any one*** question.

12. (a) Write a program in sage to find the GCD of two numbers a, b using Euclidean Algorithm.

(b) Write sage code for the following :
Find the number of digits in 2021! and compute the number of zeros and the number of ones present in 2021!. [Here 2021! means factorial of 2021.]

(c) Write your code in sage to find the root of the polynomial $f(x) = x^5 + x^4 + x^3 - x^2 + x - 1$ between -1 and 1.

5+(1+1+1)+2

13. (a) Write a program in sage to find the greatest of three given integers a, b, c.

(b) Write a program in sage to find the sum of the following series for any finite n :

$$2.3 + 4.5 + 6.7 + \cdots \ldots \ldots \ldots \ldots + 2n(2n + 1).$$

(c) Consider the following program :

```
i=0
while i < 5:
        print(i)
        i=i+1
        if i= =3:
            break
        else:
            print(0)
```

What will be the output of the program segment? 4+3+3

[Scientific Computing with R]

1. Answer ***all*** questions with proper explanation / justification (***one*** mark for correct answer and ***one*** mark for justification) : 2×10

(a) What should we type to get the value of tan (90°) in R?

(i) tan 90° (ii) $N\left(\tan\left(\frac{\pi}{2}\right)\right)$

(iii) tan(pi/2) (iv) None of these.

(b) The value of factorial of 3^{91} in R is

(i) not given (ii) factorial(3^91)

(iii) Infinity (iv) None of these.

(c) Which of the following R codes will print "Hello, world"?

(i)
```
> f <- function()  {
+              cat("Hello, world!\n")
+ }
> f()
```

(ii)
```
> f <- function()  {
+              cat("Hello, World!\n")
+ }
< f()
```

(iii)
```
> f <- function()  {
+              cat("Hello world!\n")
+ }
>>= f()
```

(iv)
```
> f <- function()  {
–              cat("Hello World!\n")
+ }
<= f()
```

(d) What will be the output of the following R code?

```
> f <- function (a, b) {
+         a^2
+ }
> f (2)
```

(i) 4 (ii) 3

(iii) 2 (iv) 5.

(e) What will be the output of the following R code snippet?

```
> paste ("a", "b", se = ":")
```

(i) "a+b" (ii) "a=b"

(iii) "ab:" (iv) None of these.

(f) The output of the following R program will be :

```
> n <- 0
> square <- 0
> while (square <= 100) {
+ n <- n + 1
+ square <- n ^ 2
+ }
> print(n)
> print(square)
```

(i) 11
100

(ii) 11
121

(iii) 11
100

(iv) None of these.

(g) What will be the output of the following R code?
options (digits = 16)
20/6

(i) 3.33

(ii) 3.333

(iii) 3.3333333

(iv) 3.3333333333333333.

(h) Which one of the following is the correct command to obtain the following matrix?

$$A = \begin{bmatrix} 20 & 25 & 30 \\ 20 & 15 & 10 \end{bmatrix}$$

(i) A <–matrix(nrow=2, ncol=3, data=c(20,20,25,15,30,10), byrow=T)

(ii) A <–matrix(nrow=3, ncol=2, data=c(20,20,25,15,30,10), byrow=T)

(iii) A <–matrix(nrow=2, ncol=3, data=c(20,25,30,20,15,10), byrow=F)

(iv) A <–matrix(nrow=2, ncol=3, data=c(20,25,30,20,15,10), byrow=T).

(i) How does a vector differ from a list?

(i) Vector and list are same and can be used interchangeably.

(ii) Vector is used only for numeric data, while list is used for any kind of data.

(iii) A vector contains item of a single data type, while a list can contain items of different data types.

(iv) Vector is like array, while list is like data frame.

(j) What is the output of the code given below?
A=10
B=20
print(A,B)

(i) 10

(ii) 20

(iii) 10 20

(iv) Error.

Section - I

Answer ***any one*** question.

2. Find the outputs of the following R commands :

(a) x=5;y=2; x=x^2+y^2; y=(x+1)/5; y

(b) x=1:7; x=x+3; x

(c) x=8; x=(x+1)**2; x=sqrt(x); x

(d) A=matrix(c(1,2,3,4), nrow=2);B=matrix(c(4,3,2,1), nrow=2)

X=A*B+2; X 2+2+2+4

3. Find the outputs of the following R commands.

(a) x=7:2 ; x=x^2+1; x

(b) x=7:2; x=x^2+2; x=length(x);x

(c) A=matrix(c(1,2,3,4), nrow=2);B=matrix(c(–4,3,–2,1), nrow=2)

X=A%*%B+2; X 3+3+4

Section - II

Answer ***any one*** question.

4. Write R code to plot the graph of the following in one plot :

(a) graph of the function f(x) =x^2+1 in range (–2,2)

(b) graph of the tangent line $f'(x)=2x$ in range (–2,2) at the point (1,2). 5+5

5. Write R code(s) for plotting

(a) the graph of polar curve

$$r = 1+2\cos(\theta),\ 0 \le \theta \le 2\pi$$

(b) the following functions in one plot :

(i) f(x) = |x|+1, in $-2 \le x \le 2$ with blue color

(ii) g(x)=sin(x) +2 cos(x), in $-2 \le x \le 2$ with red color. 4+3+3

Section - III

Answer ***any one*** question.

6. Write R code to do the following for the function $f(x) = x^3 + x$ in [–2,2] :

(a) find $f'(x)$

(b) find $f''(x)$

(c) plot $f'(x)$ and $f''(x)$ in [–2,2] with blue and green colors respectively. 3+3+2+2

7. Write R commands to find the values of the following integrals :

(a) $\int_{-\infty}^{+\infty} e^{-x^2} dx$ (b) $\int_{-\pi}^{+\pi} \sin(5x) dx$

(c) $\int_{-\pi/4}^{+\pi/4} \tan(x)dx$ (d) $\int_{-\pi/4}^{+\pi/4} e^x.\tan(x)dx$

(e) $\int_{-1}^{+1} \frac{(1+x\wedge 3)}{(1+x\wedge 2)}dx.$ 2+2+2+2+2

Section - IV

Answer ***any one*** question.

8. Without using inbuilt functions, write R code to find the mean and median of the data set
c(3,4,7,1,7,4,8,9,10,11,2,12). 5+5

9. Without using inbuilt functions, write R code to find GCD and LCM of the numbers 12 and 75. 6+4

Section - V

Answer ***any one*** question.

10. Given the matrix, $A=\begin{bmatrix} 2 & 5 & 3 \\ 4 & 5 & 2 \\ 6 & 3 & 4 \end{bmatrix}$, write R programs

(a) to find A^3

(b) to find the determinant and the inverse of A

(c) to find the inverse of A as well as to check whether the result is correct. 3+4+3

11. (a) Solve the following system of equations by a R program :

$4x + y - z = 3$
$2x + 7y + z = 19$
$x - 3y + 12z = 31.$

(b) Write a R program to find roots of the polynomial $x^7 + 3x^4 + 8x + 9$. 6+4

Section - VI

Answer ***any one*** question.

12. (a) Write a R program to print first n non-Fibonacci positive integers.

(b) Write a R program to print all the prime numbers in the interval [131,291]. 5+5

Please Turn Over

13. (a) Write a R program to find the sum of the series :

$$1^2 + 2^2 + 3^2 + \ldots + N^2.$$

(b) Discuss how we can solve the differential equation : $\frac{dx}{dt} = 1 - x$ using R commands. Also write the code for plotting the solution in a specific range. 5+5