2021

MATHEMATICS — GENERAL

Paper : SEC-B-1

(Mathematical Logic)

Full Marks : 80

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notations have their usual meanings.]

1. Choose the correct option and justify your answer : (1+1)×10

(a) The number of different non-equivalent statement formulas containing n statement letters is

(i) 2^{n^2} (ii) 2^n

(iii) 2^{2^n} (iv) 2^{2n}.

(b) A tautology statement formula is

(i) $p \wedge \sim p$ (ii) $(p \vee q) \wedge \sim (p \wedge q)$

(iii) $p \rightarrow (q \rightarrow (p \wedge q))$ (iv) $\sim(\sim p \wedge q)$.

(c) If $p \leftrightarrow q$ is logically equivalent to a statement formula A, then A may be

(i) $p \rightarrow q$ (ii) $q \rightarrow p$

(iii) $(p \rightarrow q) \vee (q \rightarrow p)$ (iv) $(p \rightarrow q) \wedge (q \rightarrow p)$.

(d) An adequate system of connectives is

(i) $\{\vee, \rightarrow\}$ (ii) $\{\wedge, \rightarrow\}$

(iii) $\{\sim, \rightarrow\}$ (iv) $\{\sim, \leftrightarrow\}$.

(e) Negation of $\exists x\,(P \rightarrow Q)$ is equivalent to

(i) $\exists x\,(\neg P \rightarrow \neg Q)$ (ii) $\forall x\,(P \wedge \sim Q)$

(iii) $\exists x\,(P \rightarrow \neg Q)$ (iv) $\forall x\,(\neg P \wedge Q)$.

(f) If Γ is a set of well formed formulas (wffs) and α and β are well formed formulas, and $\Gamma \cup \{\alpha\} \vdash \beta$ then

(i) $\Gamma \vdash \alpha$ (ii) $\Gamma \vdash (\alpha \rightarrow \beta)$

(iii) $\Gamma \vdash (\beta \rightarrow \alpha)$ (iv) $\Gamma \vdash \beta$.

(g) Let $p(x)$ be a predicate on a non-empty set D. Then $\sim \forall x\, p(x) \equiv$

(i) $\exists x \sim p(x)$ (ii) $\sim \exists x\, p(x)$

(iii) $\forall x \sim p(x)$ (iv) $\sim \exists x \sim p(x)$.

(h) Which one of the following is in prenex normal form?

(i) $\forall x\ (x < y) \rightarrow \exists z\ (x < z \wedge z < y)$ (ii) $\exists v \sim P \rightarrow \forall v\ P$

(iii) $\exists v\ (P \rightarrow Q)$ (iv) $\exists v\,(P \rightarrow Q) \leftrightarrow \big(\forall v\,(Q \rightarrow P)\big)$.

(i) In a Bengali class

(i) Bengali is the meta language (ii) Bengali is the object language

(iii) Nepali is the object language (iv) French is the object language.

(j) The inverse of the statement formula $(\sim r \rightarrow s)$ is

(i) $(r \rightarrow \sim s)$ (ii) $(\sim r \rightarrow \sim s)$

(iii) $(s \rightarrow r)$ (iv) $(s \rightarrow \sim r)$.

Unit - I

2. Answer ***any two*** questions :

(a) Let p, q, r be the propositions :

p : The date of election has been declared

q : The votes will be casted on EVM

r : The votes will be casted on ballots.

Express each of the following propositions in English language :

(i) $p \wedge q$, (ii) $p \wedge (q \wedge r)$, (iii) $p \wedge (q \vee r)$, (iv) $p \wedge (q \vee \sim r)$ 1+1+1+2

(b) Find the truth value of the statement "if x is an odd integer, then x^2 is an odd integer". Write down the converse and contra positive statement of the above statement with its truth value. 1+2+2

(c) Find the truth table of the statement formula : $\big((\sim p \rightarrow q) \vee (\sim r \rightarrow p)\big) \leftrightarrow (\sim q \wedge r)$. 5

Unit - II

3. Answer ***any six*** questions :

(a) (i) Write the following sentence as statement form, using statement letters to stand for atomic sentences.

Either Ram will come to the college and Sam will not; or Ram will not come to the college and Sam will attend all the classes.

(ii) Determine whether the following are tautologies.

$$\big(((A \Rightarrow B) \Rightarrow B) \Rightarrow A\big) \text{ and } (A \wedge \neg A).$$ 2+(2+1)

(b) Find the DNF of the formula $\sim (A \rightarrow B) \vee (\sim A \wedge C)$. 5

(c) A committee consisting of three members approves any proposal by majority vote. Each member can approve a proposal by pressing a button attached to their seats. Design a circuit as simple as you can which allow current to pass when and only when a proposal is approved. 5

(d) Determine whether the formula $\left(A \vee \left((B \wedge C)\right)\right) \rightarrow \left((A \leftrightarrow C) \vee B\right)$ is a tautology. 5

(e) Define the terms 'premises', 'consequence', 'proof' in propositional calculus. 1+2+2

(f) Let P_1, P_2, P_3 be distinct prime formulas. Find the simplest formula that is equivalent to every formula whose prime constituents are P_1, P_2, P_3 and whose corresponding truth value operation is *f*. 5

P_1	P_2	P_3	$f(P_1, P_2, P_3)$
T	T	T	T
F	T	T	F
T	F	T	T
F	F	T	F
T	T	F	T
F	T	F	T
T	F	F	T
F	F	F	T

(g) State Deduction Theorem in Propositional Calculus.
Show that $\{B \rightarrow C, C \rightarrow D\} \vdash B \rightarrow D$ where B, C, D are well formed formulas in Propositional Calculus. 2+3

(h) Show that the well formed formula $B \rightarrow \left(C \rightarrow (B \wedge C)\right)$ is a theorem in Propositional Calculus, where B, C are well formed formulas in Propositional Calculus. 5

(i) Show that each statement form in the column I is logically equivalent to the form in the column II.

	I	II
(i)	$A \Leftrightarrow B$	$(A \wedge B) \vee (\neg A \wedge \neg B)$
(ii)	$A \wedge (B \oplus C)$	$(A \wedge B) \oplus (A \wedge C)$

where $\oplus$ stands for 'Exclusive OR'. 2+3

(j) Prove that $\vdash \left(\sim B \rightarrow (B \rightarrow C)\right)$, where B, C are well formed formulas in Propositional Calculus. 5

Unit - III

4. Answer ***any four*** questions :

(a) Define atomic formula and well formed formula in Predicate Calculus. Symbolize the given sentence in a well formed formula : "Anyone who is persistent can learn Logic". 1+2+2

(b) Describe about a formal theory for Predicate Calculus. 5

(c) Define free and bound variable in a well formed formula. Find the free and bound variable(s) in the well formed formula :

$(\forall x_1)\left(A_1^2(x_1, x_2) \to (\forall x_1) A_1^1(x_1)\right).$ 3+2

(d) Test the logical validity of the given arguments : All integers are rational numbers. All rational numbers are real numbers. 2 is an integer. Therefore, 2 is a real number. 5

(e) Indicate the free and bound occurences of each variable in the following :

(i) $\left((\forall x_1)(x_1 > 0)\right) \wedge \left(\exists x_2 (x_2 = x_1)\right)$

(ii) $\exists x_1 \forall y_1 (z = y_1 \vee y_1 = x_1).$ 3+2

(f) Prove that $\vdash (\forall x)(B \leftrightarrow C) \to \left((\forall x) B \leftrightarrow (\forall x) C\right)$ where B, C are well formed formulas in Predicate Calculus. 5

(g) Reduce to prenex normal form : 5

$\forall x \forall y \left(x < y \to \exists z \left((x < z) \wedge (z < y)\right)\right).$
