2021

MATHEMATICS — HONOURS

Eighth Paper

(Module : XV)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols used in the question paper carry usual meaning.

Group - A

(Marks : 25)

Answer ***any five*** questions.

1. (a) If $\Delta(f(x)) = f(x+1) - f(x)$ and $\Delta^2(x^2 f(x)) + \Delta(2x^2 f(x)) + x^2 f(x) = 5(x+2)^2$ where $f(x)$ is defined on $-\infty < x < \infty$, find $f(x)$.

   (b) Why different interpolation formulae are used instead of a unique one? 3+2

2. (a) Establish Lagrange's polynomial interpolation formula without remainder term for interpolating $f(x)$ at the distinct points x_0, x_1, x_2, x_3.

   (b) Mention one major disadvantage of using Lagrange's interpolating polynomial over Newton's general interpolating polynomial. 4+1

3. (a) Give a geometrical significance of the Weierstrass polynomial approximation theorem.

   (b) If $f(x) = u(x)v(x)$, then show that $f[x_0, x_1] = u(x_0)v[x_0, x_1] + v(x_1)u[x_0, x_1]$ and hence deduce if $g(x) = \omega^2(x)$, then $g[x_0, x_1] = \omega[x_0, x_1][\omega(x_0) + \omega(x_1)]$. 2+(2+1)

4. What do you mean by degree of precision of a mechanical quadrature formula? For which quadrature it is maximum? Stating the error term of Simpson's one-third rule comment upon the degree of precision.

   Give geometrical interpretation of composite Simpson's $\frac{1}{3}$rd rule. 2+1+1+1

5. Show that an iterative method for computing the value of $\sqrt[k]{a}\,(a > 0)$ is given by $x_{n+1} = \frac{1}{k}\left[(k-1)x_n + x_n \frac{a}{k-1}\right]$ and also deduce that $\epsilon_{n+1} \simeq -\frac{k-1}{2\sqrt[k]{a}}\epsilon_n^{\,2}$ where ϵ_n is the error at the nth iteration. What is the order of this iterative method? 4+1

6. Show that for solving a system of linear equations by a suitable iterative method, a sufficient condition is that the system should be diagonally dominant. Is it possible that some system which is not diagonally dominant may converge to its solution by the iterative method? Justify. 3+2

7. Describe Gauss – Seidel iterative method for solving the system, $AX = b$, where A is a $n \times n$ matrix, X is an $n \times 1$ column vector and b is an $n \times 1$ column vector. How do you stop the method? 3+2

8. (a) Describe Power Method to find the dominant eigen pair of a non-singular matrix. When does this method fail?

(b) Show that the initial approximation x_0, for finding $\frac{1}{N}$, where N is a positive integer, by Newton–Raphson method must satisfy $0 < x_0 < \frac{2}{N}$, for convergence. 3+2

9. (a) Use Picard's method to compute y for $x = 0.2$, correct to 4 decimal places for the differential equation $\frac{dy}{dx} = xy + x^2$ with end condition $y(0) = 0$.

(b) Prove or disprove : The fourth order Runge–Kutte method for solving $\frac{dy}{dx} = f(x)$ with end condition $y(x_0) = y_0$ can be expressed as Simpson's $\frac{1}{3}$rd rule to the interval $[x_0, x_0 + h]$ having two equal subintervals of width $\frac{h}{2}$. 3+2

Group - B

(Marks : 25)

Section - I

Answer ***any two*** questions.

10. (a) 'A ROM is essentially a RAM'— Comment on the correctness of the statement with a justification of your opinion.

(b) Use 10's complement method subtract $(65405)_{10}$ from $(3200)_{10}$ using 5 digits computing device.

(c) If $(141)_B = (70)_{10} + (10)_8$, then find the value of B.

(d) State three essential features of an algorithm. 2+3+2+2

11. (a) Write down an efficient C or FORTRAN program to display the smallest prime number which is greater than a given positive integer.

(b) Write a C program to print all the first 100 positive integers that are divisible by 10. 4+5

12. (a) Draw a flowchart with conventional symbols to find the value of the expression,

$$\frac{mv}{t} - \left(f(v)\right)^2 + \frac{ct}{2}$$

for a given value of v and given positive values of m, c and t, where $f(y) = \begin{cases} 0{\cdot}5 & \text{if } |y| \le 1 \\ 2y^2 & \text{if } |y| > 1 \end{cases}$

(b) Design an algorithm to find the second largest element of the following list of numbers :

78, 68, 87, 82, 20, 46, 56, 23, 117, 17. 4+5

13. (a) Write a C or FORTRAN program to find a real root of the equation $x \tan^{-1}x - x^2 + 1 = 0$ in the interval [1, 2] correct to four significant figures by Newton–Raphson method.

(b) Write a C or FORTRAN program to evaluate $\displaystyle\int_{20^\circ}^{40^\circ} \frac{dx}{\left(\sqrt{2\cos x} + \sqrt{2 + \sin x}\right)^{\frac{3}{2}}}$

by Simpson's $\frac{1}{3}$rd rule with subintervals 12. 5+4

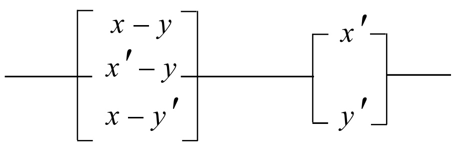
Section - II

Answer ***any one*** question.

14. (a) State the Huntington Postulates which define a Boolean Algebra (B, +, $\cdot$, $'$, 1, 0). Write down the main differences between Boolean algebra and algebra of real numbers.

(b) Does there exist a Boolean algebra with three elements? Justify your answer. 2+2+3

15. (a) Write a Boolean function for the following circuit :



(b) A committee consists of three persons. A proposal is approved by a majority of votes. One member has a voting weight 1 and each of the others has weight 2. Design a simple circuit with a bulb, so that the light will glow if and only if majority of votes is cast in favour of the proposal. 3+4