2021

PHYSICS — HONOURS

Paper : VII-A

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***any five*** questions : 2×5

(a) Show that the number of microstates available to a rigid rotor with angular momentum less than M is $\left(\frac{2\pi M}{h}\right)^2$.

(b) Three identical bosons can be distributed among two states. Show all possible configuration of the microstates.

(c) State the postulate of equal a priori probability.

(d) Draw the phase trajectory of one dimensional linear harmonic oscillator of constant energy E.

(e) The field components $E_y = E_o\sin(kx + \omega t)$ and $B_z = -B_o\sin(kx + \omega t)$ statistics Maxwell's euqations in free space. Find relation between k and ω.

(f) What is Rayleigh Scattering?

(g) Find from poynting flow, the mean value of the intensity of the magnetic field in air at a distance of 100 cm from a radiating, source of power 10 KW.

Answer ***any four*** questions.

2. (a) Show that the partition function of ideal gas consisting of N non-intereting particles of mass m is

$$Z = \left[\left(\frac{2\pi m k_B T}{h^2}\right)V\right]^N$$

where symbols have their usual meanings. Hence derive the equation of state of ideal gas from it.

(b) Derive the following relations :

(i) mean energy $\bar{E} = -\frac{\partial \ln Z}{\partial \beta}$

(ii) Pressure $\bar{p} = \frac{1}{\beta}\frac{\partial \ln Z}{\partial V}$. (4+2)+(2+2)

Please Turn Over

3. (a) A system of non-relativistic neutrons at temperature $T = 0$ has a density $10^{48}/m^3$. What is the value of the Fermi momentum for the system? (mass of Neutron is $1{\cdot}67\times10^{-27}$ kg.)

(b) Sketch the Fermi distribution function for three temperatures $T_3 > T_2 > T_1 = 0$ on the same graph.

(c) Show that the Fermi energy E_F of electrons in a metal at $T = 0$ is given by, $E_F = \frac{h^2}{2m}\left(\frac{3n}{8h}\right)^{\frac{2}{3}}$, where symbols have their usual meanings. 3+3+4

4. (a) From Einstein's theory of specific heat, show that $C_V = 3R\left(\frac{h\nu}{k_BT}\right)^2 \frac{e^{h\nu/k_BT}}{\left(e^{h\nu/k_BT}-1\right)^2}$, where symbols have their usual meanings. How does it explain the low temperature behaviour of specific heat.

(b) Evaluate the sum, $Z = \sum_{n=0}^{\infty} e^{-\beta\left(n+\frac{1}{2}\right)h\nu}$ and then calculate $U = -\frac{\partial \ln Z}{\partial \beta}$ where $\beta = \frac{1}{k_BT}$.

(6+1)+3

5. (a) Starting from Maxwell's equations, show that any initial change density in a conductor dissipates in a characteristic time.

(b) Find the average energy density for a plane monochromatic wave.

(c) Show that poynting theorem predicts Jouls heating in a wire. 3+3+4

6. (a) The refractive index for a gaseous system is given by

$$n = 1 + \frac{Nq^2}{2m\in_o}\sum_j \frac{f_j\left(\omega_j^2-\omega^2\right)}{\left(\omega_j-\omega\right)^2+\gamma_j^2\omega^2}.$$

Show that, away from resonance, it reduces to the Cauchy equation

$$n = 1 + A\left(1+\frac{B}{\lambda^2}\right),$$

the symbols have their usual meanings.

(b) What is anamolous dispersion? Find out the width of the anamolous dispersion region for a single resonance frequency ω_0. Given the damping constant $\gamma << \omega_0$. Using dimensional analysis, find out how does the intensity depend on the wavelength in Rayleigh Scattering. 2+(1+4+3)

7. (a) A metal has a conductivity 6×10^6 mhos/m. Obtain the skin depth in that metal for an electromagnetic wave of frequency 100 Hz. Deduce the formula that you use.

(b) An electromagnetic wave is incident at the surface of two linear homogeneous dielectrics. Write down the boundary conditions at the surface. Find out the conditions under which there is a phase reversal for the reflected wave.

(c) A rectangular wave guide has dimensions 2.3 cm×1.0 cm. If the driving frequency is 1.70×10^{10} Hz, find out the TE modes that would propagate through this guide. 3+4+3
