2021

PHYSICS — HONOURS

Paper : DSE-A2(b)

(Advanced Classical Dynamics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer ***any five*** questions : 2×5

(a) If $\dot{x} = rx - x^3$ is cast in the form $\dot{x} = -\frac{1}{\Gamma}\frac{dV}{dx}$, then identify $V(x)$, and plot it for $r > 0$.

(b) If I_1, I_2 and I_3 be the principal moments of inertia of a rigid body then show that $I_3 \le I_1 + I_2$.

(c) Prove that in the absence of any non-potential forces, the generalized momentum corresponding to any cyclic coordinate is a conserved quantity.

(d) Write down Hamilton's equations of motion using Poisson bracket.

(e) For a free particle moving in one-dimension, show that $x - \frac{p}{m}t$ is a conserved quantity.

(f) For a particle having a Lagrangian $L = \frac{\dot{x}^2}{2x} - V(x)$, find the Hamiltonian H.

(g) Determine the fixed points of $\dot{x} = \sin x$.

Group - B

Answer ***any three*** questions.

2. Find the geodesics on a two-dimensional flat plane in plane polar (r, θ) coordinates. 5

3. Using Fermat's principle, find the path followed by light ray in a medium whose refractive index n is proportional to $1/y$. 5

4. For a thin uniform square plate of side a and mass m, derive principal moments of inertia. Also calculate the moment of inertia about a diagonal. 3+2

5. Using the method of Lagrange's undetermined multiplier, find the dimensions of a parallelepiped of maximum volume that is circumscribed by a sphere of radius R. 5

6. Consider the dynamical system $\dot{x} = x(1-x)$. Find the fixed points. Draw the $\dot{x}$ vs. x graph and comment on the nature of the fixed points from the graph with the use of flows. Use linear stability analysis to determine the nature of any one of the fixed points. 2+2+1

Group - C

Answer ***any four*** questions.

7. (a) Consider a particle of mass m moving under the action of gravity on the surface of a cylinder which is placed in such a way that its axis is horizontal. Show that the critical angle (θ_c) at which the particle sliding off from the surface is given by $\theta_c = \cos^{-1}\left(\frac{2}{3}\right)$.

(b) A particle of mass m moves with uniform velocity $\vec{v}$ in a rotating frame whose angualr velocity (w) is constant. Write down the Lagrangian of the particle. Identify the velocity dependent potential.

(c) Given $H(x,p) = \sqrt{p^2 + m^2} + V(x)$. Find the corresponding Lagrangian. 4+4+2

8. (a) Given $S = \int_0^1 \frac{\dot{x}^2}{t^3} dt$. Extremize S to find $x(t)$ with conditions $x(t=0) = 1$ and $x(t=1) = 2$.

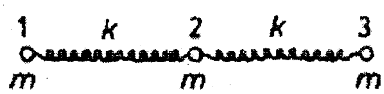
(b) If $f(q, p)$ and $g(q, p)$ are constants of motion, then show that their Poisson bracket $\{f, g\}$ is also a constant of motion.

(c) Using Poisson bracket show that $F = \frac{pq}{2} - Ht$ is a constant of motion if the Hamiltonian of the system is given by $H = \frac{p^2}{2m} - \frac{1}{2q^2}$. 4+3+3

9. (a) Consider a system of particles executing small oscillations in a conservative force field around an equilibrium point. Show that the expression of potential energy, when expressed in terms of normal coordinates, assume the form of homogeneous quadratic functions.

(b) A particle of mass m is moving in a potential $V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$ where a and b are positive constants. Find the frequency of small oscillations about a point of stable equilibrium.

(c) Three particles of equal masses m move without friction in one dimension. Two end particles are each connected to the third by a massless spring of spring constant k. Write down the Lagrangian of the system. Form the secular equation and hence derive the normal-mode angular frequencies of the normal modes of the system. 3+3+(1+2+1)



10. (a) Express kinetic energy of a rigid body in terms of its component particles. Hence define moment of inertia tensor from this expression.

(b) Find the moments and products of inertia of a uniform square plate of side '*a*' and mass '*m*' about the *z*-axis where *z* axis is perpendicular to the plane of the plate and passes through the centre of the plate. 5+5

11. (a) Consider the differential equation

$$\ddot{x} + 2b\dot{x} + \omega^2 x = f \cos \Omega t$$

(i) Cast it in the non-autonomous dynamical system form.

(ii) What is the dimension of the dynamical system?

(iii) How would you convert this non-autonomous system into an autonomous one?

(iv) What would be the dimensionality of the autonomous system?

(v) Is the autonomous system conservative or dissipative?

(b) For the model

$$\dot{x} = x(a - by)$$
$$\dot{y} = y(cx - d)$$

where a, b, c, d are all positive constants find out the fixed points of the system. Using the stability matrix identify the nature of the fixed points. (1+1+1+1+1)+(2+3)

12. (a) For the map

$$x_{n+1} = 3{\cdot}3\ x_n(1 - x_n)$$

(i) What are the fixed points?

(ii) Determine their nature.

(b) For $\dot{X} = r + X^2$

draw $\dot{X}$ vs. X for

(i) $r = 1$, $r = 0$ and $r = -1$.

(ii) For each of the above three cases determine the number of fixed points.

(iii) What is flow around the fixed point for the $r = 0$ case? (2+2)+(3+2+1)