2021

MATHEMATICS — HONOURS

Paper : DSE-A(2)-3

(Fluid Statics and Elementary Fluid Dynamics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***all*** questions with proper explanation/justification (***one*** mark for correct answer and ***one*** mark for justification) : 2×10

(a) Falling drops of water become spherical due to

(i) Cohesion (ii) Adhesion

(iii) Viscosity (iv) Surface tension.

(b) Which of the following is incorrect?

(i) Density has the units of mass per unit volume.

(ii) Specific weight has units of weight per unit volume.

(iii) Shear stress has the units of force per unit area.

(iv) Specific gravity has the units of mass per unit volume.

(c) Which principle explain the buoyant force of a fluid?

(i) Archimedes principle (ii) Pascal's principle

(iii) Bernoulli's principle (iv) Newton's second law of motion.

(d) Which of the following substances has the highest viscosity?

(i) Air (ii) Hydrogen

(iii) Mercury (iv) Water.

(e) The position of centre of pressure on a plane surface immersed vertically in a static mass of fluid is

(i) at the centroid of the submerged area.

(ii) always above the centroid of the area.

(iii) always below the centroid of the area.

(iv) none of the above.

(f) A floating body is said to be in a state of stable equilibrium

(i) when its metacentric height is zero.

(ii) when metacentre is above the centre of gravity.

(iii) when metacentre is below the centre of gravity.

(iv) only when its centre of gravity is below its centre of buoyancy.

(g) The horizontal component of thrust on a curved surface is equal to the

(i) product of pressure, intensity at its centroid and area.

(ii) force on a vertical projection of the curved surface.

(iii) weight of the liquid vertically above the curved surface.

(iv) force on the horizontal projection of the curved surface.

(h) Which of the following velocity potentials satisfies continuity equation?

(i) x^2y (ii) $x^2 - y^2$

(iii) $\cos x$ (iv) $x^2 + y^2$.

(i) The continuity equation is derived using the concept of

(i) Conservation of mass (ii) Conservation of linear momentum

(iii) Conservation of angular momentum (iv) Conservation of energy.

(j) The three components of velocity in a velocity field are given by

$$u = Ax + By + Cz,\ v = Dx + Ey + Fz,\ w = Gx + Hy + Jz.$$

The relationship among the coefficients A through J that is necessary if this is to be a possible incompressible flow field is given by

(i) $A + D + J = 0$ (ii) $B + F + H = 0$

(iii) $C + G + E = 0$ (iv) $A + E + J = 0$.

Unit - 1

2. Answer ***any one*** question :

(a) Distinguish between ideal fluids and real fluids. Explain the importance of compressibility in fluid flow. 5

(b) Derive the fundamental fluid static equation in scalar form and hence obtain the hydrostatic equation $\frac{\partial p}{\partial z} = -\rho g$ (symbols having usual meaning). 5

Unit - 2

3. Answer ***any two*** questions :

(a) (i) An area is bounded by two concentric semi-circles with common bounding diameter in the surface of a liquid. Show that the depth of its centre of pressure is $\frac{3\pi}{16}\frac{(a+b)\left(a^2+b^2\right)}{a^2+ab+b^2}$ where a and b are the radii of the semi-circles.

(ii) Prove that the centre of pressure (C.P.) of a completely submerged plane surface in a fluid under the action of gravity is always below the centre of gravity of the submerged surface. 5+5

(b) (i) A triangular lamina is wholly immersed in a liquid with its vertex in the surface and its base horizontal. Show how to draw a horizontal line to divide it into two parts, the thrusts on which are equal.

(ii) A closed right circular cone is filled with liquid and held with its axis vertical and vertex upwards. Find the resultant vertical and horizontal thrusts on half the curved surface determined by any plane through the axis. 4+6

(c) Explain the term metacentre of a floating body. For a body floating freely in a homogeneous fluid at rest under gravity, prove that $HM = \frac{AK^2}{V}$, where the symbols have their usual meanings. 3+7

(d) (i) What do you understand by convective equilibrium? Establish the relation $\frac{T}{T_0} = 1 - \frac{\gamma-1}{\gamma}\frac{z}{H}$ for the atmosphere in convective equilibrium of temperature and statisfying the law $p = k\rho^{\gamma}$, where T = Absolute temperature at height z, T_0 = Absolute temperature at sea level, h = Height of homogeneous atmosphere (gravity is assumed to be constant).

(ii) A box is filled with a heavy gas at a uniform temperature. Prove that if a is the altitude of the highest point above the lowest, and p, p' are the pressures at these two points, the ratio of the pressure to the density at any point is equal to $\frac{ag}{\log \frac{p'}{p}}$. (1+4)+5

Unit - 3

4. Answer ***any one*** question :

(a) (i) The velocity components for a two-dimensional flow system are given by

$$u = 2x + 2y + 3t,\ v = x + y + \frac{1}{2}t.$$

Find the displacement of a fluid particle in the Lagrangian system, where the initial value of (x, y) is (x_0, y_0).

(ii) What is meant by the material derivative of a fluid property? Mention the two parts of the material derivative of the fluid property. 6+4

(b) (i) Establish the relation between Eulerian and Lagrangian description of fluid flow.

(ii) The velocity vector in a fluid flow is given by $\vec{v} = 2x^3\hat{i} - 5x^2y\hat{j} + 4t\hat{k}$. Is the fluid flow steady? Calculate the velocity and acceleration of the fluid particle at (1, 2, 3) at time $t = 1$. 5+(1+1+3)

Unit - 4

5. Answer ***any two*** questions :

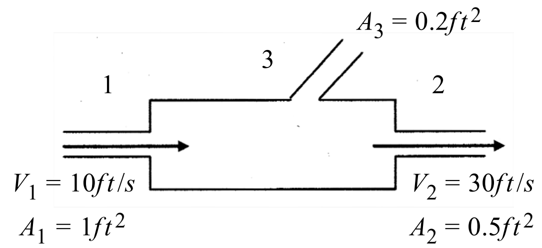
(a) Derive the continuity equation of compressible flow of a fluid in rectangular Cartesian system and hence find it for a steady incompressible fluid flow. 4+1

(b) (i) The velocity components in a two-dimensional flow are $u = 8x^2y - \frac{8}{3}y^3$, $v = -8xy^3 + \frac{8}{3}x^3$.

Show that these velocity components represent a possible case of an incompressible flow.

(ii) Derive momentum equation from Reynold's transport theorem for fixed control volume. 2+3

(c) Consider steady, incompressible flow through the device shown in the figure below. Determine the magnitude and the direction of the volume flow rate through the port 3. 5



(In the above figure V's and A's are velocities across the ports and cross-sections of them respectively)

(d) Derive the general energy equation for a control volume system. 5