2021

MATHEMATICS — GENERAL

Paper : DSE-B-2

(Mathematical Finance)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

(Marks : 10)

1. Choose the correct alternative : 1×10

(a) If the amount P is borrowed for t years at a nominal interest rate of r percent per year compounded continuously, then the amount owed at time t is

(i) $P(1 + r)^t$ (ii) Pe^{rt}

(iii) Pe^{2rt} (iv) None of these.

(b) Suppose that you borrow the amount P, to be repaid after one year along with interest at a rate r percent per year compounded semi-annually. How much is owed in a year?

(i) $P(1 + r)^2$ (ii) $P\left(1+\frac{r}{2}\right)^2$

(iii) $P(1 + 2r)^2$ (iv) None of these.

(c) The money an investor receives for taking on a risk is called

(i) risk premium (ii) arbitrage

(iii) option value (iv) risk-free rate.

(d) According to residual dividend policy, a firm should pay a dividend of all left over when

(i) zero NPV projects have been funded

(ii) positive NPV projects have been funded

(iii) projects with IRR equal to risk-free interest rate have been funded

(iv) projects with IRR greater than risk-free interest rate have been funded.

(e) If the co-variance between stock A and market returns is 15, and the standard deviation of market return is 3 then what is the value of beta?

(i) 1.66 (ii) 1.67

(iii) 5.0 (iv) None of these.

(f) The price of a stock is ₹ 1,000, and there are 40% chances that it would be ₹ 950 and 60% chances that it would be ₹ 1,150 the next year. What is the percentage of expected return?

(i) 7.5% (ii) 7.0%

(iii) 8.0% (iv) 10.0%

(g) What is the real rate of interest if nominal rate is 10% and inflation rate is 4%?

(i) 5.7% (ii) 5.8%

(iii) 5.6% (iv) 3.8%

(h) If a loan is started with nominal interest rate 8%, then the effective interest rate will be

(i) 8.16% (ii) 8.10%

(iii) 8.20% (iv) 8.00%

(i) The normalized version of covariance is called

(i) regression (ii) correlation

(iii) cross-section (iv) spread.

(j) The measure for calculating how much two random variables change together is called

(i) variance (ii) covariance

(iii) skewness (iv) kurtosis.

Group - B

(Marks : 15)

Answer ***any three*** questions.

2. Many credit-card companies charge interest at a yearly rate of 18% compounded monthly. If the amount P is charged at the beginning of a year, how much is owed at the end of the year if no previous payments have been made? Also, if the amount ₹ 10,000 is charged at the beginning of the year, determine the amount that is owed at the end of the year. 3+2

3. What do you mean by expected return and standard deviation? Give a suitable example to explain them. What is the difference between them? 2+1+2

4. Let $D(t)$ denote the amount you would have on deposit at time t if you deposit D at time 0 and interest is continuously compounded at rate r. Show that, for h small, $D(t + h) \approx D(t) + rhD(t)$. Also, establish $D(t) = De^{rt}$. 3+2

5. State and prove Arbitrage Theorem. 1+4

6. Consider a portfolio comprising of three securities in the following proportions and with the indicated security beta.

Security	*Amount Invested*	*Beta*	*Expected Return*
A	₹ 1.5 L	1.0	12.0%
B	₹ 1.0 L	1.5	13.5%
C	₹ 2.0 L	0.8	9.0%

(i) What is the portfolio's beta?

(ii) What is the portfolio's expected return? 3+2

Group - C

(Marks : 40)

Answer ***any four*** questions.

7. (a) Mr. Amitava plans to retire in 20 years has decided to put an amount A in the bank at the beginning of each of the next 240 months, after which he will withdraw ₹ 10,000 at the beginning of each of the following 360 months. Assuming a nominal yearly interest rate of 6% compounded monthly, how large does A need to be?

(b) Find the yield curve and the present value function if $r(s) = \frac{1}{1+s} r_1 + \frac{s}{1+s} r_2$, where r denotes the interest rate at time s and r_1 and r_2 are two constants. 6+4

8. (a) When a function $f(x)$ is said to be convex?

(b) Let $C(K, t)$ be the cost of a call option on a specified security that has strike price K and expiration time t. Show that for fixed expiration time t, $C(K, t)$ is a convex and nonincreasing function of K. Also, show that $C(K, t) - C(K + s, t) \le se^{-rt}$, for $s > 0$. 2+(4+4)

9. (a) Describe the method of bisection to find an approximate value of a real root of the equation $f(x) = 0$.

(b) An investor who pays CF_0 to buy a bond that will pay coupon interest CF_1 after one year and CF_2 (coupon interest plus face value) after two years. The investor wants to find the internal rate of return or yield to maturity that solves the equation $CF_0 = \frac{CF_1}{1+IRR} + \frac{CF_2}{(1+IRR)^2}$. Find the internal rate of return by taking $CF_0 = 90$, $CF_1 = 10$, $CF_2 = 100$. 5+5

10. (a) State the basic assumptions behind the Markowitz portfolio theory.

(b) What is portfolio diagram?

(c) Derive the expressions for portfolio mean return and variance. 3+3+(2+2)

11. (a) Find the correlation coefficient between X, Y where $2X - 3Y + 1 = 0$.

(b) An investor with capital x can invest any amount between 0 and x; if y is invested then y is either won or lost, with respective probabilities p and $1 - p$. If $p > \frac{1}{2}$, how much should be invested by an investor having a log utility function? 4+6

12. State Markowitz mean-variance problem. To solve this problem set the Lagrangian function. Give an outline to optimize this function. 3+2+5

13. What do you mean by conditional value at risk or CVAR? If the gain G from an investment is a normal random variable with mean μ and standard deviation σ, then calculate the CVAR. 2+8
