# 2021

## MATHEMATICS — HONOURS

**Fifth Paper**

**(Module - X)**

**Full Marks : 50**

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*(Notations and symbols have their usual meaning.)*

## Group-A

**(Marks-20)**

### Section-I

**(Linear Algebra-II)**

Answer ***any one*** question. 10×1

1. (a) Let $V$ and $W$ be vector spaces of finite dimension over a field $F$ and $T : V \to W$ be a linear mapping. Then show that the rank of $T$ = the rank of matrix $T$. 5

   (b) A mapping $F : \mathbb{R}^3 \to \mathbb{R}^3$ maps the vector (2, 1, 1), (1, 2, 1) and (1, 1, 2) to (1, 1, –1), (1, –1, 1) and (1, 0, 0) respectively. Show that $F$ is not an isomorphism. 5

2. (a) A linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^4$ is defined by $T(x, y, z) = (y + z, z + x, x + y, x + y + z), (x, y, z) \in \mathbb{R}^3$. Find Im $T$ and dimension of Im $T$. 5

   (b) A linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(0,1,1) = (1,0,1)$, $T(1,0,1) = (2,3,4)$, $T(1,1,0) = (1,2,3)$. Find the matrix of $T$ relative to the order basis $(\epsilon_1, \epsilon_2, \epsilon_3)$ where $\epsilon_1 = (1,0,0)$, $\epsilon_2 = (0,1,0)$, $\epsilon_3 = (0,0,1)$. Deduce that $T$ is invertible. 5

### Section-II

**(Modern Algebra-III)**

Answer ***any one*** question. 10×1

3. (a) Prove or disprove : A subgroup $H$ of a group $G$ is a normal subgroup if and only if every right coset of $H$ is also a left coset. 4

   (b) Let $G$ be a group. Let $H$ be a subgroup of $G$ such that $H \subseteq Z(G)$. Show that if $G/H$ is cyclic then $G = Z(G)$, where $Z(G)$ denotes the centre of $G$. 3

   (c) Prove that the quotient group $(Q/Z, +)$ is infinite but each of its elements is of finite order. 3

**4.** (a) Suppose that there is a homomorphism from a group $G$ on $Z_{10}$. Prove that G has normal subgroups of index 2 and 5. 3

(b) If $K$ is a subgroup of $G$ and $N$ is a normal subgroup of $G$, prove that $K/(K \cap N)$ is isomorphic to $KN/N$. 3

(c) Find all homomorphism from $Z_6$ into $Z_4$. How many of those are epimorphism? Justify your answer. 2+2

## Group-B

## (Tensor Calculus)

## (Marks-15)

Answer ***any three*** questions. 5×3

**5.** If $A^i (i = 1,2, \dots, n)$ are components of an arbitrary contravariant vector and $C_{ij} A^i A^j$ is an invariant then prove that $C_{ij} + C_{ji} (i, j = 1,2 \dots, n)$ are components of a second order tensor of type (0, 2). 5

**6.** If $A_{ij} (i, j = 1,2 \dots, n)$ are components of a skew-symmetric tensor of rank 2, then prove that

$$\left(\delta^i_j\, \delta^k_l + \delta^i_l\, \delta^k_j\right) A_{ik} = 0.$$ 5

**7.** If all the components of a tensor are zero at a point in one co-ordinate system, then prove that they are all zero at this point in every co-ordinate system. 5

**8.** If $A^{ij}$ is a skew-symmetric tensor, then show that

$$A^{ij}, j = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} \left(\sqrt{g}\, A^{ij}\right).$$ 5

**9.** Find $g$ and $g^{ij}$ corresponding to the line element

$ds^2 = 3(dx^1)^2 + 2(dx^2)^2 + 4(dx^3)^2 - 6(dx^1)\,(dx^3)$ in Riemannian space $V_3$. 5

Answer either **Group-C** or **Group-D.**

## Group-C

## (Differential Equation-II)

## (Marks-15)

Answer ***any one*** question. 15×1

**10.** (a) State the first shifting property of Laplace transformation. Using this property, find the Laplace transform of $e^{-2t}(3\cos 6t - 5\sin 6t)$. 1+4

(b) Find the inverse Laplace transform of $\frac{4p+5}{(p-1)^2(p+2)}$. 5

(c) Find the power series solution of the initial value problem

$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 2y = 0,\ y(0) = 1,\ y^1(0) = 0.$ 5

**11.** (a) Find the Laplace transform of $t^2 e^{at} \sin at$. 5

(b) Using shifting property of Inverse Laplace Transform, evaluate $L^{-1}\left\{\frac{6p-4}{p^2-4p+20}\right\}$. 5

(c) Solve by using Laplace transform of $\frac{d^2y}{dt^2} + 9y = \cos 2t$, when $y(0) = 1$ and $y\left(\frac{\pi}{2}\right) = -1$. 5

## Group-D

## (Graph Theory)

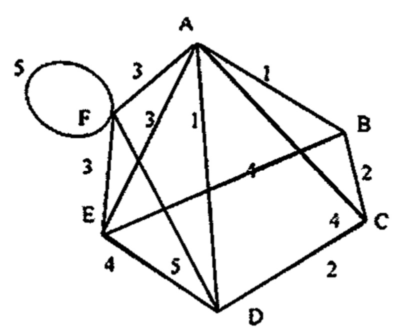
## (Marks-15)

Answer ***any three*** questions. 5×3

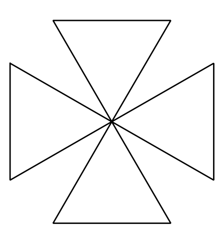
**12.** (a) Show that there is no simple graph with six vertices of which the degrees of five vertices are 5, 5, 3, 2 and 1. 2

(b) Prove that the number of odd degree vertices of a graph *G* is always even. 3

**13.** Obtain a minimal spanning tree of the following graph using Kruskal's algorithm. 5




**14.** (a) Find a Euler trail in the following graph *G*. 3



(b) Explain spanning tree in a simple connected graph with example. 2

**15.** State and prove the necessary and sufficient condition for a graph to be an Euler graph. 5

**16.** (a) Show that a complete graph with $n$ vertices consists of $^{n(n-1)}/_{2}$ edges. 3

(b) Prove that a connected graph with $n$ vertices and $(n-1)$ edges is a tree. 2