2021

PHYSICS — HONOURS

Paper : CC-13

(Electromagnetic Theory)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Find the velocity of light in a medium for which relative permittivity and relative permeability are 3 and 2 respectively.

(b) Define a good and a bad conductor from the point of view of the frequency of the incident electromagnetic wave.

(c) Show that the intrinsic impedance for a plane wave in the linear isotropic medium is $\sqrt{\mu/\epsilon}$.

(d) What do you mean by Evanescent wave?

(e) In a 2-dimensional anisotropic medium, the electric displacement $\vec{D}$ is given by $\vec{D} = \frac{3}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$. If the permittivity tensor is given by $\epsilon = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, find the electric field $\vec{E}$.

(f) Find the state of polarization of the following light beams :

(i) $\vec{E} = \hat{i}\, a\sin(\omega t - kz) + \hat{j}\, a\cos(\omega t - kz)$

(ii) $\vec{E} = \hat{i}\, a\sin(\omega t - kz) + \hat{j}\, b\sin(\omega t - kz)$.

(g) Calculate the thickness of a half-wave plate for a light of wavelength 500 nm.

[Given : $n_o = 1{\cdot}5442$; $n_e = 1{\cdot}5533$]

2. (a) Show how Maxwell's equations in free space imply local conservation of charge. Show that this implies Kirchoff's second law.

(b) Consider the electromagnetic potential $U = e\phi - \vec{V}.\vec{A}$, where ϕ is the scalar potential and $\vec{A}$ is the vector potential. Show that under gauge transformation, U transforms as a total derivative.

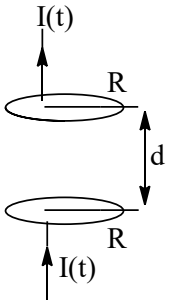
(c) Given the Electric field $\vec{E} = E_o\left(\hat{j} + a\hat{k}\right)e^{ik_o\left[-ct+(y+\sqrt{3}z)\right]}$ where a is a constant. Find out

(i) a.

(ii) velocity of the wave

(iii) refractive index of the medium. (2+1)+3+(1+2+1)

3. Consider circular capacitor plates parallel to each other and having radius R, total charge Q. They are separated by a distance d. Current I(t) is flowing as shown in figure.



(a) Find the stored total energy density u in the region between the plates. (You can assume $\vec{E}$ field to be uniform, neglect fringing effects).

(b) Calculate the induced magnetic field ($\vec{B}$).

(c) Find Poynting vector ($\vec{S}$) and show its direction.

(d) Show that $\frac{\partial u}{\partial t} + \bar{\nabla}\cdot\bar{S} = 0$. 2+2+3+3

4. (a) Show that in a conductor, the electric and magnetic fields are not in phase.

(b) Show that if a monochromatic linearly polarized plane wave is moving in an isotropic non-conducting medium, the time average of its energy density is distributed equally between the magnetic and electric fields.

(c) What is Skin Depth? Find the phase velocity and the magnitude of attenuation constant of plane wave at a frequency of 10 GHz in polyethelene.

(Given : $\mu = \mu_0$; $\epsilon_r = 2{\cdot}3$; $\sigma = 2{\cdot}56\times10^{-4}\ S/m$) 3+4+(1+2)

5. (a) What is Brewster's angle?

(b) An incident wave along $\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}$ falls on a refractive surface at $z = 0$. If the refractive index is $\mu = \sqrt{3}$, find the propagation vectors for the reflected and the refracted rays.

(c) A plane electromagnetic wave falls obliquely on air-glass interface. Find the angle of incidence for which the reflection and transmission co-efficients are each equal to 0·3.

(d) Show that the frequency of the wave remains unchanged upon refraction, when an electromagnetic wave is incident on the plane interface between two different media. 2+3+3+2

6. (a) Explain how to distinguish the following :

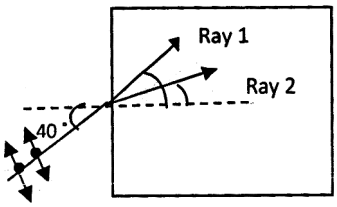
(i) circularly polarized

(ii) elliptically polarized

(iii) mixture of elliptically polarized and unpolarized light.

(b) Explain how continuous variation in retardance of a wave plate can be achieved.

(c) An negative crystal is cut so that the optic axis is perpendicular to the plane of the paper. Given $n_o = 1{\cdot}66$, $n_e = 1{\cdot}50$.



(i) State with reason which is the E and O ray out of the two rays.

(ii) Determine the angle between the two refracted rays.

(d) Two nicol prisms are so arranged that no light is transmitted through them. The analyzer is now rotated through 30º and then through another 45º. The transmitted intensities were noted in each case. which of the two intensities will be greater than the other and by what percentage?

3+2+(1+2)+2

7. (a) A left circularly polarized light propagating along z direction falls on a half-wave plate made from calcite crystal. Optic axis of the plate is cut parallel to the surface. Write down x and y components of electric field after the wave emerges out of the plate. What is the state of polarization of the emergent light?

(b) The specific rotation of light with $\lambda = 5893$ Å in quartz is 21·7º. Find the difference in the refractive indices between the two circular motions in the medium.

(c) Explain Fresnel's theory of rotation of plane of polarization by an optically active substance. A plane polarized light is found to rotate 12º due to propagation through 20 cm polarimeter tube. If the specific rotation of the solution is 60º $dm^{-1}g^{-1}cm^{-3}$, find the concentration of the solution.

3+2+(3+2)