2021

PHYSICS — HONOURS

Fifth Paper

Full Marks : 100

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***any ten*** questions : 2×10

(a) Is the constraint given by $x\dot{x} + y\dot{y} + x\dot{y} + \dot{x}y = k$ (a constant), a holonomic constraint?

(b) Show that the two Lagrangians $L_1 = (q - \dot{q})^2$ and $L_2 = \left(q^2 + \dot{q}^2\right)$ are equivalent.

(c) Prove that for motion of a particle under central force, the areal velocity with respect to the centre of force remains constant.

(d) If the kinetic energy $T = \frac{1}{2} m \dot{r}^2$ and the potential energy $V = \frac{1}{r}\left(1 + \frac{\dot{r}^2}{c^2}\right)$, find the Hamiltonian '$H$' and determine whether $H = T + V$.

(e) Explain what is meant by streamlines.

(f) Derive the equation of continuity for a compressible fluid.

(g) For a four vector A^{μ} show that $A_{\mu} A^{\mu}$ is a scalar.

(h) Find the constant C which makes $e^{-\alpha x^2}$ an eigenstate of the operator. $\frac{d^2}{dx^2} - Ex^2$ (α is a constant).

(i) Can we measure the kinetic and potential energies of a particle simultaneously with arbitrary precision?

(j) Why are the Stokes lines brighter than anti-Stokes lines in Raman Spectra?

(k) The electronic configuration of Mg is $1s^2\ 2s^2\ 2p^6\ 3s^2$. Obtain its spectral term.

(l) Why is pure vibrational spectra observed in liquid?

Group - A

Section - I

(Classical Mechanics II)

Answer ***any two*** questions.

2. (a) Starting from Lagrange's equation of motion, obtain Hamilton's equation of motion using Legendre transformation.

(b) For the Hamiltonian H = $q_1p_1 - q_2p_2 - aq_1^2 + bq_2^2$, solve the Hamilton's equation of motion and prove that q_1q_2 = constant and $\frac{(p_2 - bq_2)}{q_1}$ = constant.

(c) Show that the effective potential of a particle of mass 'm' in a central force field is given by $U_{eff}(r) = U(r) + \frac{L^2}{2mr^2}$, where L is the angular momentum. 4+3+3

3. (a) Consider a simple harmonic oscillator with angular frequency ω_0. What will be its angular frequency when a constant force K is applied on it?

(b) The point of suspension of a simple pendulum moves simple harmonically along the vertical line. Obtain the Lagrangian of the system.

(c) Prove that, if the Lagrangian of an unconstrained system is invariant under continuous translation, then the total linear momentum is conserved. 3+4+3

4. (a) State Bernoulli's equation of fluid motion and mention the conditions of its validity.

(b) The Lagrangian of a particle of mass m is $L = \frac{1}{2}\left(m\dot{x}^2 - bx^2\right)e^{at}$ where a and b are positive constants. Determine the Hamiltonian. Is it a constant of motion?

(c) A flat vertical plate is struck normally by a horizontal jet of water 50 mm in diameter with a velocity of 18 m/s. Calculate the force on the plate assuming it to be stationary. 3+4+3

Section - II

(Special Theory of Relativity)

Answer ***any two*** questions.

5. (a) Define the interval between two events in space time. Show that it is invariant under a Lorentz transformation. Hence explain the conditions for which the interval is time-like, space-like or light-like.

(b) A muon at rest has life time 2×10^{-6} sec. What is its life time when it travels with a velocity $\frac{3}{5}$c ?

(c) Define covariant and contravariant vector. (1+2+3)+2+2

6. (a) Discuss about inconsistency, if any, in Newton's law of gravitation in the light of postulates of special theory of relativity.

(b) Define Minkowski space. Show that Lorentz transformation can be regarded as transformation due to a rotation of axes through an imaginary angle given by $\theta = tan^{-1}(i\beta)$ where $\beta = \frac{v}{c}$ in the 4-dimensional Minkowski space.

(c) Two rods of proper length l_0 move lengthwise towards each other parallel to the common axis with the same velocity v relative to the laboratory frame. Show that the length of each rod in the reference frame fixed to the other rod is $l = l_0 \frac{(1-\beta^2)}{(1+\beta^2)}$, $\beta = \frac{v}{c}$. 2+(1+3)+4

7. (a) Define proper time interval $d\tau$. Hence construct velocity four vector. Show that it is a time-like vector.

(b) If $A^{\mu\nu}$ and $B^{\mu\nu}$ are two tensors, Show that $A^{\mu\nu} B_{\mu\nu} = A_{\mu\nu} B^{\mu\nu}$.

(c) For two four vectors A and B, prove that $A_\mu B^\mu = A^\mu B_\mu$. 4+4+2

Group - B

Section - I

(Quantum Mechanics II)

Answer ***any two*** questions.

8. (a) Consider a one-dimensional simple harmonic oscillator moving in a potential $V(x) = \frac{1}{2} m\omega^2 x^2$.

Given that the ground state wave function is $\psi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} \exp\left(-\frac{1}{2}\alpha x^2\right)$ (where $\alpha = m\omega/\hbar$). Find the expectation value of (x^2).

(b) For a Hamiltonian $\hat{H} = (\hat{p}^2/2m) + V(\hat{x})$, prove that $\left[\hat{x}, \left[\hat{x}, \hat{H}\right]\right] = -\frac{\hbar^2}{m}$.

(c) Prove that $\exp\left[i(\hat{A}\hat{B} - \hat{B}\hat{A})\right]$ is a Hermitian operator, if $\hat{A}, \hat{B}$ are Hermitian operators. 4+3+3

9. (a) A stream of particles of mass m and energy E move towards the potential step $V(x) = 0$ for $x < 0$ and $V(x) = V_0$ for $x \geq 0$. If the energy of the particles $E < V_0$,

(i) show that there is a finite probability of finding the particles in the region $x > 0$.

(ii) sketch the solutions in the two regions.

(iii) determine the reflection coefficient and comment on the result.

(b) Write down Pauli's spin matrices σ_x, σ_y and σ_z. The eigenfunctions of the Pauli spin operator σ_z are α and β. Show that $\frac{\alpha+\beta}{\sqrt{2}}$ and $\frac{\alpha-\beta}{\sqrt{2}}$ are the eigenfunctions of σ_x. (3+1+2)+(2+2)

10. (a) Write down the Schrödinger equation for the hydrogen atom assuming the nucleus heavy. Obtain the radial part of the equation.

(b) In the ground state of hydrogen atom show that the probability P for the electron to lie within a sphere of radius R is

$$P = 1 - \exp\left(-\frac{2R}{a_0}\right)\left(1 + \frac{2R}{a_0} + 2R^2/a_0^2\right) \text{ where } \Psi(100) = (\pi a_0^3)^{-1/2} \exp(-r/a_0).$$

(c) Write down the operators for L^2 and L_z in polar coordinates. Hence verify that $\Psi = A \sin\theta\, e^{i\phi}$, where A is a constant, is an eigenfunction of L^2 and L_z. Find the eigenvalues. 4+2+4

Section - II

(Atomic Physics)

Answer ***any two*** questions.

11. (a) In a Stern–Gerlach experiment, a beam of silver atoms moving with a velocity 'v' passes through an inhomogeneous magnetic field of gradient $\frac{\partial B}{\partial z}$ for a distance of 'l'. After emerging from the magnetic field, they travel a distance 'b' before reaching the screen. What will be the magnitude of the splitting?

(b) What is the g-factor for an atom with a single optical electron in $d_{\frac{3}{2}}$ level?

(c) Consider the L-S coupling scheme for helium atom. Show that (i) $1s^1 2s^1$ configuration leads to the terms $^1S_0$ and $^3S_1$ while (ii) $1s^1 2p^1$ configuration leads to $^1P_1$, $^3P_0$, $^3P_1$ and $^3P_2$.

4+2+(2+2)

12. (a) The spacing between the vibrational levels of CO molecule is 0.08 eV. Calculate the value of the force constant of the CO bond. Given that the masses of C and O atoms are 2.0×10^{-26} kg and 2.7×10^{-26} kg respectively. ($\hbar = 6.58 \times 10^{-16}$ eV sec)

(b) Do hydrogen molecules give rise to pure vibration-rotation spectra? Justify your answer.

(c) Pure rotational spectrum is almost always seen as absorption lines, and not as emission lines. Explain. 4+3+3

13. (a) Draw the energy level diagram for a four-level laser. Explain the requirement of each energy level. Why is a four-level laser perferred to a three-level laser?

(b) In a He-Ne laser transition from $3S$ to $2P$ level gives a laser emission of wavelength 632.8 nm. If the $2P$ level has energy equal to $15.2 \times 10^{-19} J$, assuming no loss, calculate the pumping energy required.

(c) Why do molecules show band spectra rather than line spectra? (2+3+1)+2+2
